Questions for self-checking

1. Give definitions of the electric circuit, electric current, electromotive force, potential, voltage, power, work.

2. Give definitions of passive (resistance, inductance, capacitance) and active (voltage source, current source) elements of an electric circuit. Draw diagrams and volt-ampere characteristics of the ideal and real voltage source and current source.

3. Give definitions of dual elements, give examples of dual circuits, dual laws of electric circuit.

Problems

1. Build (qualitatively) time dependences of a current through a linear capacitance for the cases, when the voltage on the capacity is such as shown in Fig. 1.11, *a*, *b*.

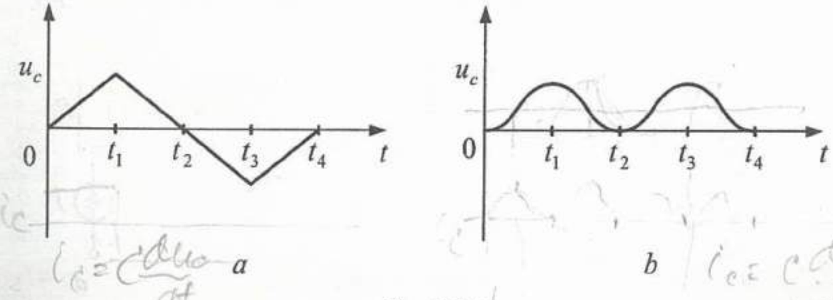


Fig. 1.11

2. In Fig. 1.12, *a*, *b* are given time dependences of the instantaneous power consumption of these circuits that do not contain sources of energy. Where do the elements of this circuit store energy?

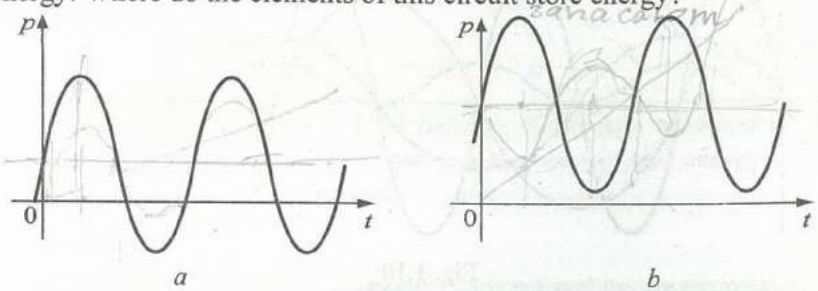


Fig. 1.12

3. In Fig. 1.13, *a*, *b* are shown time dependences of the energy supplied to circuits that do not contain sources of energy. Do these circuit elements contain an energy accumulator? Is there enough data to answer this question?

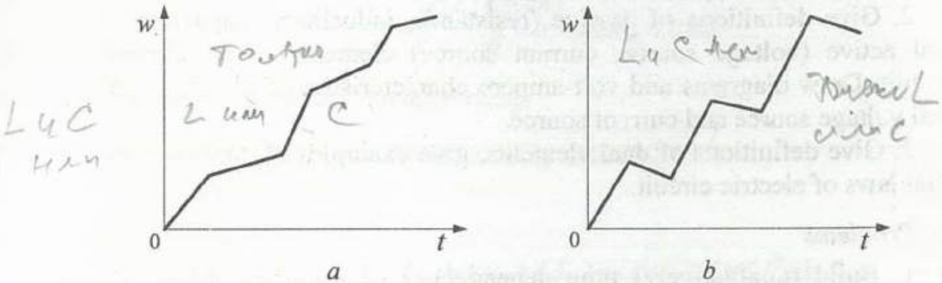


Fig. 1.13

4. Write an analytical expression for the external characteristics of the sources of electric energy whose circuits are shown in Fig. 1.14, *a-d*.

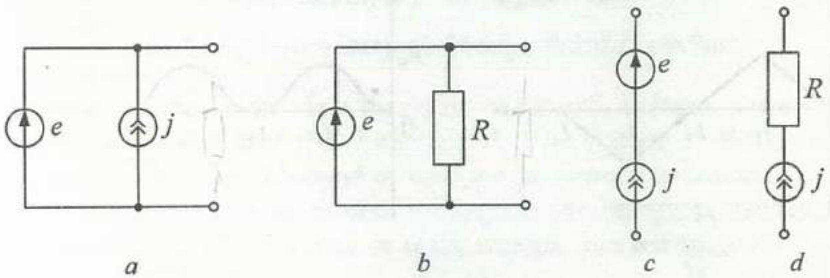


Fig. 1.14

5. The supplied voltage — $U_1 = 9{,}38$ V; the load resistance — $R_{l1} = 1\text{M}\ \Omega$.

When reducing the resistance of the load to $R_{l1} = 0{,}1\ \text{M}\Omega$, the source voltage reduces to U_2= 6,78 V. Determine the parameters of the elements for the series and parallel equivalent circuits of the source.

2. *Questions for self-checking*

1. Give definitions of the main topological concepts (branch, node, path, loop, tree, edge, chord, section). Specify these concepts in the electric diagram.

2. State the rules for constructing the matrix of incidences (nodes), the matrix of loops, and the matrix of cross-sections. How are the main loop and its direction determined? How are the main section and its direction determined?

Problems

1. Construct a graph of the electric circuit whose diagram is shown in Fig. 2.6.

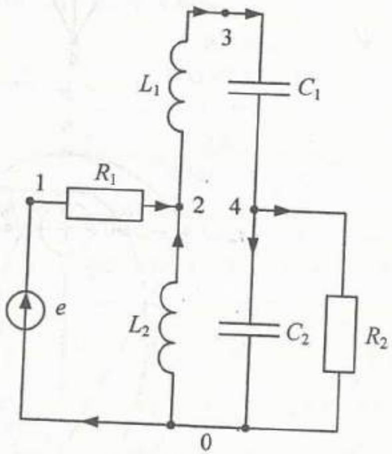


Fig. 2.6

2. Construct a graph of the electric circuit for the incidence matrix:

$$A=\begin{vmatrix} -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 & 1 \end{vmatrix}.$$

3. Build several different trees of the graph in Fig. 2.7. Point out the system of main loops, corresponding to each tree.

4. Derive a matrix of the main sections of the graph shown in Fig. 2.7. Specify the tree corresponding to this section and composed of branches 1, 2, 4, [illegible].

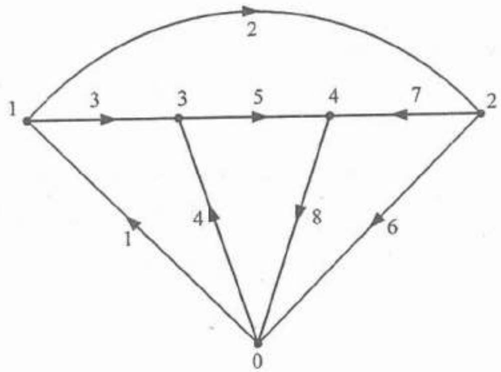


Fig. 2.7

5. Derive a matrix of the main loops of the circuit using the initial data of problem 4.

3. *Questions for self-checking*

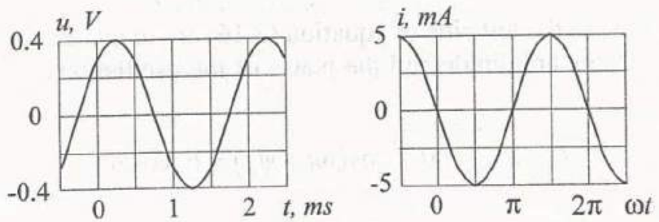
1. Give the definition of a harmonic current, identify the major notations of harmonic current, point out its basic properties. Give the definitions of the root means square (RMS) and average values of a harmonic current.

2. What forms of complex number representation do you know? What is the complex amplitude of a harmonic current?

3. Give the formulas for the direct and inverse *K*-transformations. How are the image of the derivative of harmonic current and the image of the integral of harmonic current to be calculated in terms of complex amplitudes?

Problems

1. Determine the amplitude, RMS, period, frequency, angular frequency and the initial phase of a harmonic voltage and current, the plots of which are shown in Fig. 3.4, *a*, *b*.



a *b*

Fig. 3.4

2. Find the amplitude, RMS, frequency, angular frequency, and the initial phase of the following harmonic voltages:

$$u_1 = 5\cos\left(10^4 t + 60^\circ\right)\text{V};\; u_2 = 5\sqrt{2}\cos\left(2\pi 10^4 t - 60^\circ\right)\text{V};$$

$$u_3 = 4,24\sin\left(100\pi t + \frac{\pi}{6}\right)\text{V}.$$

3. Determine the complex amplitudes, complex RMS and instantaneous complexes functions, as shown in Fig. 3.4.

4. For the given complex RMS or the complex amplitudes of voltage find the instantaneous values of the voltage. The angular frequency of all voltages is the same. Construct diagrams of the voltages:

$$\dot{U}_{m1} = 7,5e^{j\frac{\pi}{4}}\text{V};\; \dot{U}_{m2} = 0,56e^{124^\circ}\text{V};$$

$$\dot{U}_{m3} = 50\text{mV};\; \dot{U}_{m4} = 3\times10^{-3}e^{j45^\circ}\text{V};$$

$$\dot{U}_{m5} = 48\sqrt{2}\text{V};\; \dot{U}_{m6} = 14.7 + j16,3\text{V};$$

$$\dot{U}_{m7} = 14,7 - j16,3\text{V}; \dot{U}_{m8} = -14,7 + j16,3\text{V};$$

$$\dot{U}_{m9} = -14,7 - j16,3\text{V};\; \dot{U}_{m10} = 707 - j0,121\text{V};$$

$$\dot{U}_{m11} = 6,6\times10^{-8} + j2,42\times10^{-4}\text{V};\; \dot{U}_{m12} = 3 + j0,15\text{V}.$$

5. Calculate the complex amplitudes of currents defined by the relationships:

a) $\dot{I}_{m1} = \dot{I}_{m2} + \dot{I}_{m3}$; b) $\dot{I}_{m2} - \dot{I}_{m3}$; c) $\dot{I}_{m1} + \dot{I}_{m4} + \dot{I}_{m5} + \dot{I}_{m6} + \dot{I}_{m7}$,

where

$$\dot{I}_{m2} = 2,17e^{j47^\circ}\text{A};$$

$$\dot{I}_{m3} = 8,54e^{j168^\circ}\text{A};$$

$$\dot{I}_{m4} = 0,71e^{j54.7^\circ}\text{A};$$

$$\dot{I}_{m5} = 7,31e^{-j5.02^\circ}\text{A};$$

$$\dot{I}_{m6} = 3,32e^{j145.6^\circ}\text{A};$$

$$\dot{I}_{m7} = 5,27e^{j200.2^\circ}\text{A}.$$

4. *Questions for self-checking*

1. What is the essence of the complex amplitude method? Present the procedure for harmonic current circuit calculation by the complex amplitude method.

2. What are the complex impedance and complex admittance of a circuit? Write the expressions for the complex, active, reactive and total impedances, the expressions for the complex, active, reactive and total admittances of a circuit, and the expressions for the phase shift between the currents and voltages.

3. What are complex, total, active, and reactive powers. Write the expression for the phase shift between the current and voltage in a circuit and the power factor in terms of active, reactive and full powers.

4. Draw vector diagrams for impedances, admittances and powers in a circuit. Point out the impedance triangle, the admittance triangle, and the power triangle.

5. Name the criteria for power transfer from the power source to the load. Write the condition of maximum active power transfer from the power source to the load in a harmonic current circuit. Write the condition of maximum efficiency in the load.

6. What does the power balance condition lie in? Write the expression for the power balance condition.

7. Build vector diagrams of currents and voltages separately for resistance, inductance and capacitance, and point out the phase relationships in these diagrams.

Problems

1. Calculate the complex input impedance and admittance of a circuit the current and voltage at the input of which are:

a) $i = 7{,}07 \cdot 10^{-3} \cos\left(10^3 t + \frac{\pi}{3}\right)$ A; $u = 14{,}14 \cos\left(10^3 t + \frac{\pi}{2}\right)$ V;

b) $i = 0{,}282 \cos(100\pi + 60^\circ)$ A; $u = 50 \cos 100\pi t$ V;
$i = 5\cos(3140t + 90^\circ)$ mkA; $u = 0{,}4\cos(3140t + 45^\circ)$ V;

c) $i = 2{,}8\cos(1885t + 164^\circ)$ mkA; $u = 0{,}24\cos(1885t + 74^\circ)$ mV.

2. Calculate the resistive and reactive components of the complex input impedance of a circuit if $Y_j = 44 - j18$ mSm; $Y_2 = j0{,}12$ Sm; $Y_3 = (29 + j51) \times 10^{-4}$ Sm; $Y_4 = 15 \times 10^{-3} e^{j54^\circ}$ Sm.

3. The voltage $u = 100 \cos \omega t$ V is applied to the terminals of a circuit consisting of a resistance $R = 40\ \Omega$ and an inductance $L = 0{,}24$ mH connected in series. Determine the complex input impedance Z_{inp} and the complex amplitude of the current I_m if the stimulus frequency is $f_1 = 20$ Hz.

4. Find the complex amplitudes of voltages on the elements of the circuit spoken about in Problem 3. Build a vector diagram for the frequency.

5. In a series RC-circuit the RMS values of voltages are known: $U_R = 0{,}707$ V; $U_{inp} = 1$ V. Determine the time constant of the circuit $\tau = RC$, if the angular frequency of the stimulus is $\omega = 10\ \mathrm{s}^{-1}$.

6. Determine the complex RMS values of branch currents and voltages on the elements of the circuit. Construct a vector diagram of the currents and voltages. The diagram of the circuit is shown in Fig. 4.18; the parameters of its elements are: $R = 1$ kΩ; $L = 2$ mH; $C = 2$ nF; $E_1 = E_2 = 1$ V; $E_3 = 2$ V; $J = 2$ mA; $\omega = 10\ \mathrm{s}^{-1}$.

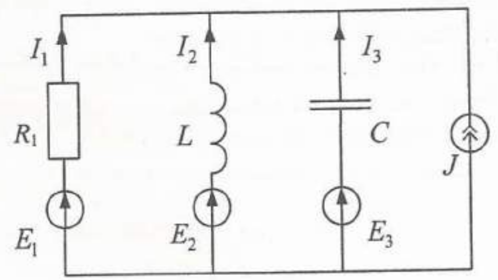


Fig. 4.18

Solution

For the circuit in Fig. 5.11, *a*:

$$C_{12} = C_1 + C_2 = 70 + 30 = 100 \text{ pF};$$

$$\frac{1}{C_{567}} = \frac{1}{C_5} + \frac{1}{C_6} + \frac{1}{C_7} = \frac{1}{300} + \frac{1}{300} + \frac{1}{300} = \frac{1}{100};$$

$$C_{567} = 100 \text{ pF};$$

$$C_{4567} = C_4 + C_{567} = 100 + 100 = 200 \text{ pF};$$

$$\frac{1}{C} = \frac{1}{C_{12}} + \frac{1}{C_{4567}} + \frac{1}{C_3} = \frac{1}{100} + \frac{1}{100} + \frac{1}{200} = \frac{5}{250} = \frac{1}{50};$$

$$C = 50 \text{ pF};$$

$$Z_{inp} = \frac{1}{\omega C_{inp}} = \frac{1}{2\pi \cdot 10^6 \cdot 50 \cdot 10^{-12}} = 3{,}185 \text{ k}\Omega.$$

For the circuit in Fig. 5.11, *b*:

$$\frac{1}{L_{1234}} = \frac{1}{L_1} + \frac{1}{L_1} + \frac{1}{L_1} + \frac{1}{L_1} = \frac{4}{8};$$

$$L_{56} = L_5 + L_6 = 8 + 8 = 16 \text{ mH};$$

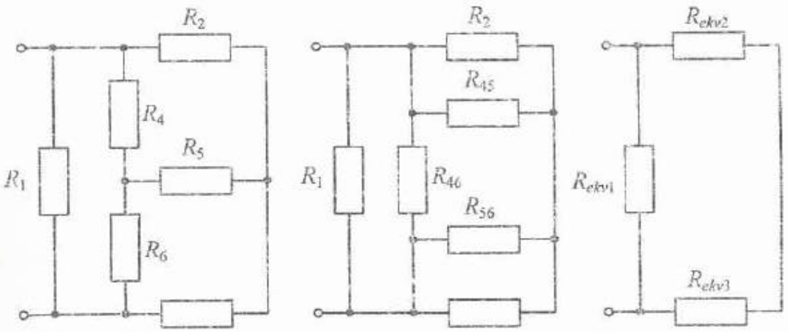
$$\frac{1}{L_{inp}} = \frac{1}{L_{1234}} + \frac{1}{L_{56}} = \frac{1}{2} + \frac{1}{16} = \frac{9}{16} \text{ mH}^{-1};$$

$$L_{inp} = 1{,}78 \text{ mH};$$

$$Z_{inp} = \omega L_{inp} = 2\pi f L_{inp} = 2\pi \cdot 10^6 \cdot 1{,}78 \cdot 10^{-3} = 11{,}2 \text{ k}\Omega.$$

Example 2

Find the input impedance of the circuit (Fig. 5.12, *a*) by means of successive equivalent transformations. The parameters of the circuit elements are: $R_4 = R_5 = R_6 = R = 4$ kΩ; $R_{45} = R_{56} = R_{64} = 2$ kΩ.



a *b* *c*

Fig. 5.12

Solution

Replace the star of the resistances R_4, R_5, R_6 by the triangle of the resistances R_{45}, R_{56}, R_{64} (Fig. 5.12, *b*):

$$R_{45} = \frac{R_4R_5 + R_5R_6 + R_6R_4}{R_6} = \frac{R^2}{R_6} = \frac{2\cdot 2 + 2\cdot 4 + 4\cdot 2}{4} = \frac{20}{4} = 5 \text{ k}\Omega;$$

$$R_{56} = \frac{R^2}{R_4} = \frac{20}{4} = 5 \text{ k}\Omega;$$

$$R_{64} = \frac{R^2}{R_5} = \frac{20}{4} = 5 \text{ k}\Omega.$$

Replace the parallel-connected resistances R_1 and R_{46}, R_2 and R_{45}, R_{34} and R_{56}, by the resistances R_{ek1}, R_{ek2}, R_{ek3} respectively (Fig. 5.12, c)

$$R_{ek1} = \frac{R_1R_{46}}{R_1 + R_{46}} = \frac{4\cdot 10}{4+10} = 2{,}86 \text{ k}\Omega;$$

$$R_{ek2} = \frac{R_1R_{45}}{R_1 + R_{45}} = \frac{4\cdot 5}{4+5} = 2{,}22 \text{ k}\Omega;$$

$$R_{ek3} = \frac{R_1R_{56}}{R_1 + R_{56}} = \frac{2\cdot 10}{2+10} = 1{,}67 \text{ k}\Omega.$$

The input impedance of the circuit is

$$R_{inp} = \frac{R_{ek1}(R_{ek2}+R_{ek3})}{R_{ek1}+R_{ek2}+R_{ek3}} = \frac{2,86(2,22+1,67)}{2,86+2,22+1,67} = 1,65\ \text{k}\Omega.$$

Example 3

Determine the current I_3 by means of equivalent transformations of the current and voltage sources (Fig. 5.13, *a*). The parameters of the circuit elements are:

$R_1 = 1\ \text{k}\Omega$; $R_2 = 2\ \text{k}\Omega$; $R_3 = 3\ \text{k}\Omega$;

$E_1 = 20$ V; $E_2 = 10$ V; $J_1 = 10$ mA; $J = 20$ mA.

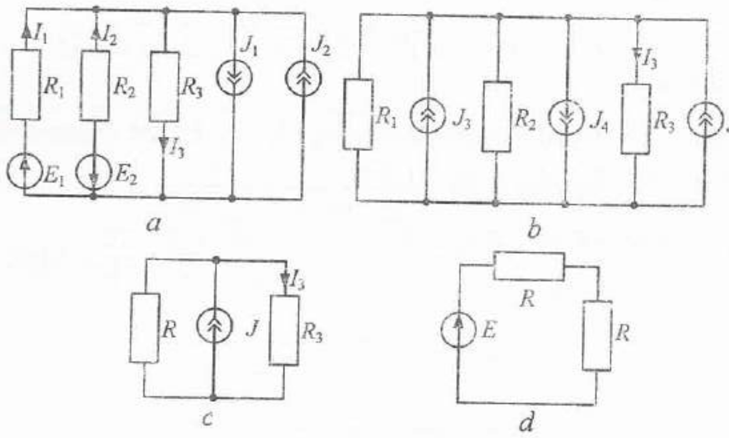


Fig. 5.13

Solution

Transform the real voltage sources E_1, R_1 and E_2, R_2 to the real current sources J_3, R_1 and J_4, R_2, and replace the ideal current sources J_1, J_2 by one equivalent current source J_5 (Fig. 5.13, *b*).

$$J_3 = \frac{E_1}{R_1} = \frac{20}{1} = 20\ \text{mA};\ J_4 = \frac{E_2}{R_2} = \frac{10}{2} = 5\ \text{mA};$$

$$J_5 = J_2 - J_1 = 20 - 10 = 10\ \text{mA}.$$

Replace the ideal current sources J_3, J_4, J_5 by one equivalent source J, and the resistances R_1, R_2 — by one resistance R (Fig. 5.13, *c*).

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{1\cdot 2}{1+2} = 0,67\ \text{k}\Omega;$$

$$J = J_3 - J_4 + J_5 = 20 - 5 + 10 = 25\ \text{mA}.$$

Replace the real current source J, R by the real voltage source E (Fig. 5.13, *d*).

$$E = JR = 25\cdot 0,67 = 16,67\ \text{V}.$$

The current now is

$$I_3 = \frac{E}{R+R_3} = \frac{16,67}{6,67+3} = 4,54\ \text{mA}.$$

Questions for self-checking

1. Write the expressions for Ohm's and Kirchhoff's laws in real, complex and matrix forms.
2. Write the relationships for equivalent transformations of a harmonic current circuit for the cases of series and parallel connections of the resistance, inductance, capacitance, voltage source and current sources. What warnings must be taken into account for the sources of energy?
3. Write the expressions for the equivalent "delta"-to-"star" transformation and vice versa.
4. Specify the rules of transformation of a voltage source into a current source and vice versa in electric circuits.

Problems

1. Calculate the complex input impedance of the circuit (Fig. 5.14) for the frequencies: $f_1 = 39,8$ kHz; $f_2 = 79,6$ kHz; $f_3 = 159$ kHz.

The parameters of the circuit elements are:

$R_1 = R_2 = 1\ \text{k}\Omega$;

$C_1 = C_2 = 0,5$ nF; $L = 10$ mH.

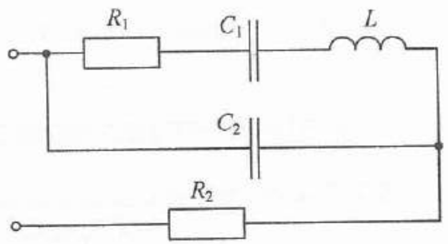


Fig. 5.14

2. Find the capacities C_{13}, C_{23}, C_{12}, for which the circuit in Fig. 5.15, *b* is equivalent to the circuit in Fig. 5.15, *a*, if $C_1 = C_2 = 340$ pF; $C_3 = 20$ pF.

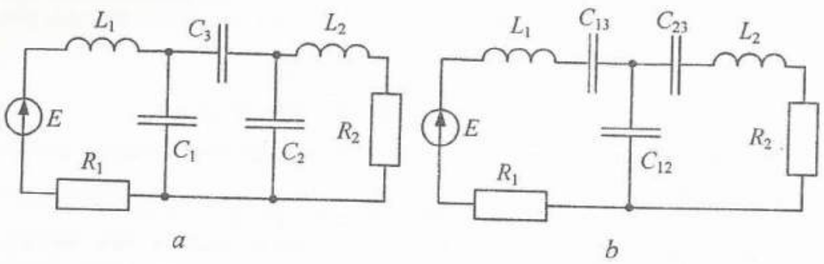


Fig. 5.15

3. Calculate the input impedance of the circuit (Fig. 5.16). The parameters of the circuit elements are:

$$R_1 = R_3 = R_6 = 10\ \text{k}\Omega;\ R_2 = R_4 = R_5 = R_7 = 2\ \text{k}\Omega.$$

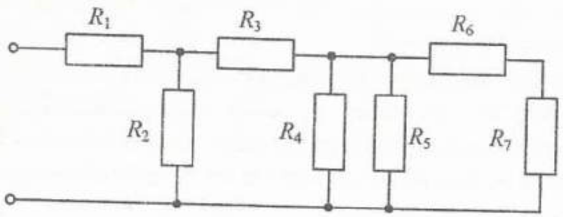


Fig. 5.16

4. Build series and parallel diagrams of the circuit in Problem 2.

5. Build a series equivalent circuit for a part of the circuit consisting of the parallel-connected resistance $R = 100\ \text{k}\Omega$ and capacitance $C = 1\ \text{nF}$ for the frequency $\omega = 10^6\ \text{s}^{-1}$..

Example 1

Calculate the frequency at which the reactive component of the input complex impedance of the circuit (Fig. 6.11) is equal to zero.

The parameters of the circuit: $L = 0{,}1\,\text{H}$; $C = 0{,}2$ mcF; $R = 2\text{k}0$ hms.

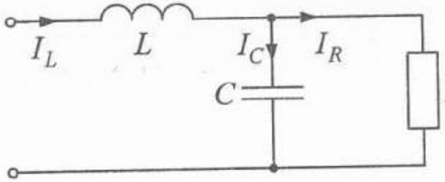


Fig. 6.11

Solution

Find the complex impedance of the circuit. The complex admittance of a parallel-connected resistance and a capacitance is

$$Y_{RC} = Y_R + Y_C = \frac{1}{R} + j\omega C.$$

The complex input impedance

$$Z_{inp} = Z_L + \frac{1}{Y_{RC}} = j\omega L + \frac{1}{\dfrac{1}{R} + j\omega C} = j\omega L + \frac{R}{1 + j\omega C} =$$

$$= j\omega L + \frac{R(1 - j\omega RC)}{(1 + j\omega RC)(1 - j\omega RC)} = jvL + \frac{R - jvR^2C}{1 + (\omega RC)^2} =$$

$$= \frac{R}{1 + (\omega RC)^2} + j\left[\omega L - \frac{vR^2C}{1 + (\omega RC)^2}\right].$$

The reactive component of the complex input impedance

$$X_{inp} = \omega L - \frac{\omega R^2 C}{1 + (\omega RC)^2}.$$

For $X_{inp} = 0$, we obtain

$$\omega L - \frac{\omega R^2 C}{1 - (\omega RC)^2} = 0; \; L - \frac{R^2 C}{1 + (\omega RC)^2} = 0.$$

Hence

$$\omega = \sqrt{\frac{1}{LC} - \frac{1}{R^2C^2}} = \sqrt{\frac{1}{0{,}1 \cdot 0{,}2 \cdot 10^{-6}} - \frac{1}{(2 \cdot 10^3)^2 (0{,}2 \cdot 10^{-6})^2}} = 6{,}6 \cdot 10^3 \text{s}^{-1}.$$

Questions for self-checking

1. Draw a harmonic current circuit with a series connection of resistance, inductance and capacitance, and write the expression for the current in the circuit and for the voltage on the elements. Build a vector diagram for the currents and voltages in the circuit where the inductive reactance is more capacitive and inductive reactance is less capacitive. Mark the resistance triangles, specify the phase angles.

2. Draw a harmonic current circuit with a parallel connection of resistance, inductance and capacitance, and write the expression for the current in the elements and for the voltage of the circuit. Build a vector diagram of the currents and voltage in the circuit where the inductive conductance is more capacitive and inductive conductance is less capacitive. Mark the conductance triangles, specify the phase angles.

Problems

1. Make all possible equilibrium equations for currents for the circuit in Fig. 6.12, write the basic system of electrical equilibrium equations for this circuit.

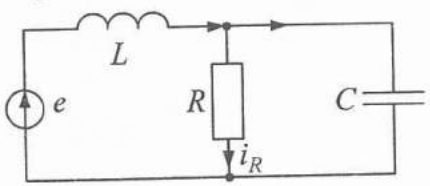


Fig. 6.12

2. In a series *RL*-circuit the RMS voltages on its elements are known: $U_R = 5\text{V}$; $U_L = 3{,}12\text{V}$. Calculate the RMS value of the voltage at the input of the circuit and the phase shift between the input current and voltage $\varphi = \psi_4 - \psi_i$.

3. Find the complex input impedance and admittance of the circuit consisting of a parallel-connected resistance $R = 40\ \Omega$ and an inductance $L = 240\ \mu\text{H}$ at the frequency $f = 120$ kHz.

4. Two series-connected circuits R_1L_1, R_2L_2, and an inductance L_3 are connected in parallel to the two terminals. Find the complex input impedance of the circuit if:

$$R_1 = 100\ \Omega; R_2 = 10\ \Omega; L_1 = L_2 = L_3\ 1\ \text{mH}; \omega = 10^4\ \text{s}^{-1}.$$

5. To the circuit described in problem 4 is connected a current source $j = 2\cos\left(126\cdot 10^3 t - 37{,}14°\right)$. Calculate the complex amplitudes of the voltages on the elements of the circuit and the currents through the elements. Construct a vector diagram.

Example 1

Set up a system of equations using the Kirchhoff equation method for the electric circuit shown in Fig. 7.8.

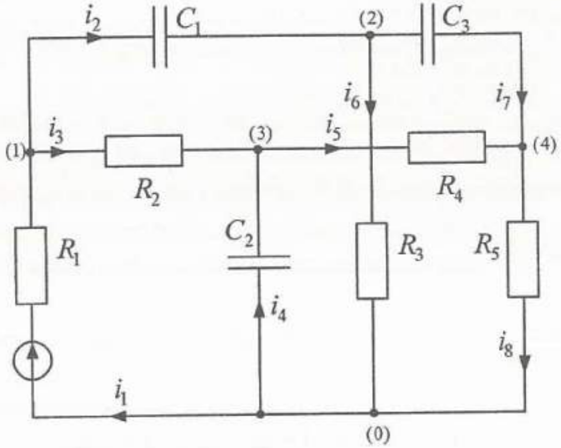


Fig. 7.8

Solution

The circuit has five nodes $(q=5)$, node 0 being the basic, and the other four – independent. The four equations are set up in accordance with Kirchhoff's current law:

for node 1: $I_1 - I_2 - I_3 = 0$;

for node 2: $I_2 - I_6 - I_7 = 0$;

for node 3: $I_3 - I_4 - I_5 = 0$;

for node 4: $I_5 + I_7 - I_8 = 0$.

The circuit consists of eight branches $(p=8)$. Then the number of independent loops is $p-q+1=4$. The other four equations are set up

according to Kirchhoff's voltage law, taking account of the chosen loop tracing directions.

For the loop $E-R_1-R_2-C_2$: $-E+I_1R_1+I_3R_2-I_4\dfrac{1}{j\omega C_2}=0.$

For the loop $C_2-R_4-R_5$: $I_4\dfrac{1}{j\omega C_2}+I_5R_4+I_8R_5=0.$

For the loop $E-R_1-C_1-R_3$: $-E+I_1R_1-I_2\dfrac{1}{j\omega C_2}+I_6R_6=0.$

For the loop $R_3-C_3-R_5$: $-I_6R_3+I_7\dfrac{1}{j\omega C_3}+I_8R_5=0.$

These eight equations constitute the full (consistent) system of equations for the circuit.

Questions for self-checking

1. Name the peculiar features of the harmonic current circuit as compared to the *DC*-circuit. Define the general procedure for obtaining equivalent complex circuits and the procedure for harmonic current circuit calculation by the complex amplitude method.
2. Define the general procedure for electric circuit calculation by the Kirchhoff equation method.
3. Define the general procedure for electric circuit calculation by the loop current method.
4. Define the general procedure for electric circuit calculation by the node voltage method.

Problems

1. Set up a system of electrical equilibrium equations for the electric circuit whose diagram is shown in Fig. 7.8.
2. Set up a system of electrical equilibrium equations for the electric circuit (Fig. 7.8) using the loop current method.
3. Set up a system of electrical equilibrium equations for the electric circuit (Fig. 7.8) using the node voltage method.
4. For the circuit (Fig. 7.9) set up a system of electrical equilibrium equations using the node voltage method.
5. Determine the currents in the branches (Fig. 7.10) using the node voltage method. The parameters of the circuit elements are:

$R_1 = R_3 = 1\,k\Omega;\ R_2 = 2\,k\Omega;\ R_4 = 0{,}8\,k\Omega;\ R_5 = 4\,k\Omega;$

$R_6 = 5\,k\Omega;\ C_1 = C_3 = 1\text{ nF}; C_2 = 2\text{ nF}; L_1 = 1\text{ mH};$

$L_2 = 4\text{ mH}; E_1 = 10\text{ V};\ J = e^{-j60^\circ}\text{ mA};\ \omega = 0{,}2\cdot 10^6\text{ s}^{-1};\ E_2 = 2\text{ V}.$

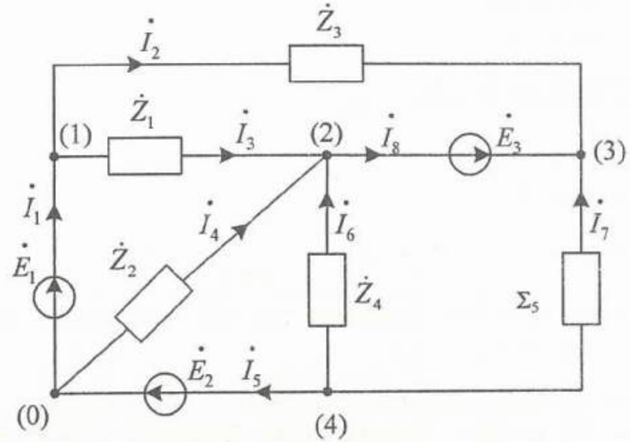


Fig. 7.9

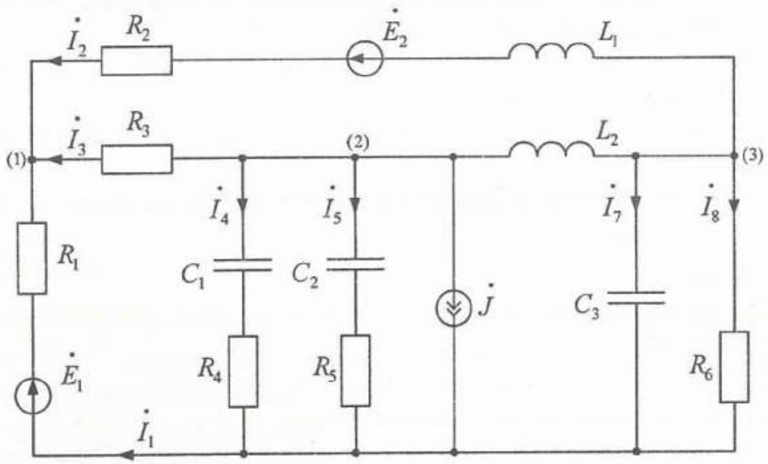


Fig. 7.10

6. For the circuit (Fig. 7.11) set up a system of electrical equilibrium equations using the node voltage method.

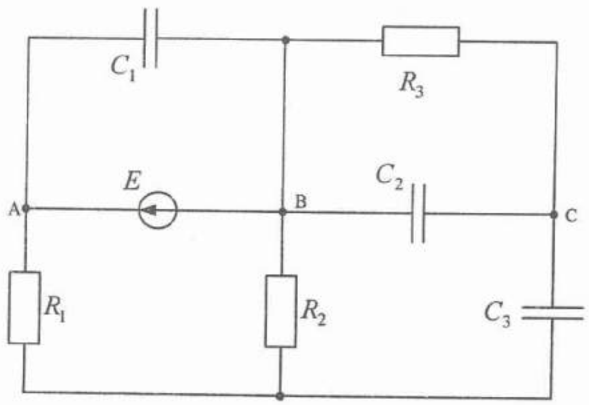


Fig. 7.11

Example 1

Determine the current in the circuit (Fig. 8.8, *a*) using the superposition theorem. The parameters of the circuit elements are: $R_1 = 6\ \Omega$; $R_2 = 4\ \Omega$; $R_3 = 12\ \Omega$; $E_1 = 120$ V; $E_2 = 100$ V.

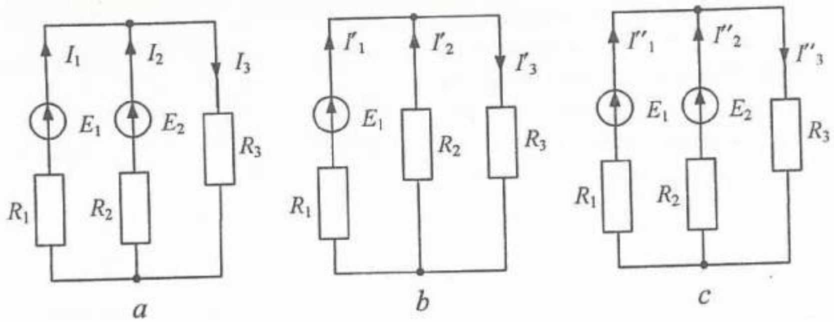


Fig. 8.8

Solution

In accordance with the superposition theorem, the current I_3 can be found as the sum of the partial currents I_3' and I_3'', flowing in the same branch under the action of the sources E_1 and E_2 separately. The circuits in which the currents I_3' and I_3'' are to be determined are presented in Fig. 8.8, *b*, *c*:

$$I_3' = \frac{E_1}{R_1 + \frac{R_2 R_3}{R_2 + R_3}} \cdot \frac{R_2}{R_2 + R_3} = \frac{120}{6 + \frac{4 \cdot 12}{4+12}} \cdot \frac{4}{4+12} = 3{,}33 \text{ A};$$

$$I_3'' = \frac{E_2}{R_2 + \frac{R_1 R_3}{R_1 + R_3}} \cdot \frac{R_1}{R_1 + R_3} = \frac{100}{4 + \frac{6 \cdot 12}{6+12}} \cdot \frac{6}{6+12} = 4{,}17 \text{ A};$$

$$I_3 = I_3' + I_3'' = 3{,}33 + 4{,}17 = 7{,}5 \text{ A}.$$

Questions for self-checking

1. State the superposition theorem. Does it hold for any circuit? Determine the procedure for circuit calculation using the superposition principle.

2. State the equivalent voltage generator theorem (Thevenin's theorem) and the equivalent current generator theorem (Norton's theorem). Determine the procedure for circuit calculation by the equivalent generator method.

3. State the reciprocity theorem. Does it hold for any circuit? State the compensation theorem. Describe the concept of Tellegen's theorem.

Problems

1. Calculate the voltages on the elements of the circuit (Fig. 8.9), using the superposition method.

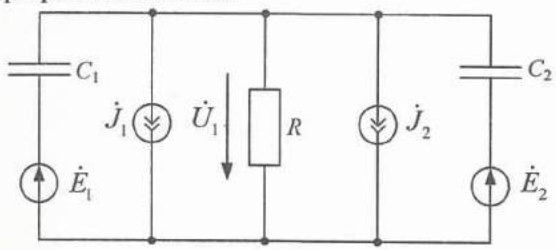


Fig. 8.9

The parameters of the circuit elements are: $R = 1\ k\Omega$; $C_1 = C_2 = 1$ pF; $J_1 = S_1 E_1$; $J_2 = S_2 E_2$; $S_1 = 1{,}5$ mSm; $S_2 = 1$ mSm; $E_1 = 0{,}1$ V; $E_2 = 0{,}2$ V; $f = 200$ MHz.

2. Determine the currents in the branches (Fig. 8.10) using the superposition method. Using the obtained results, show that the circuit is mutual. The parameters of the circuit elements are:

$R_1 = 12\ k\Omega$; $R_2 = 10\ k\Omega$; $R_3 = 0{,}2\ k\Omega$; $L_1 = 0{,}5$ mH; $L_2 = 12$ mH; $E_1 = 80$ V; $E_2 = 120$ V; $\omega = 5 \cdot 10^6\ \text{s}^{-1}$.

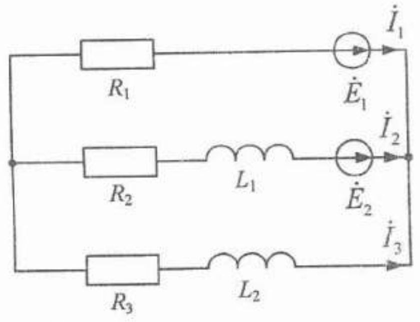


Fig. 8.10

3. Find the current in the secondary winding of an ideal transformer (Fig. 8.11) using the superposition principle and the reciprocity theorem. The parameters of the circuit elements are: $R_1 = 100\ \Omega$; $R_2 = 10\ k\Omega$; $C_1 = 5$ nF; $C_2 = 2$ nF; $L = 4$ mH; $n = 0,2$; $E_1 = 0,1$ V; $E_2 = 2$ V; $\omega = 0,4 \cdot 10^6\ s^{-1}$.

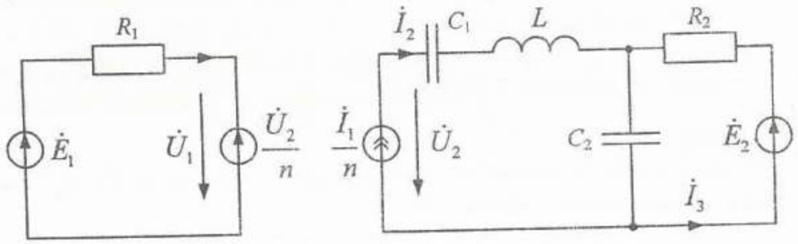


Fig. 8.11

4. Using the superposition principle and the theorem of reciprocity, determine the currents I_1, I_2 in the circuit as discussed in Example 1.

5. Determine the current (Fig. 8.12) using Thevenin's theorem.

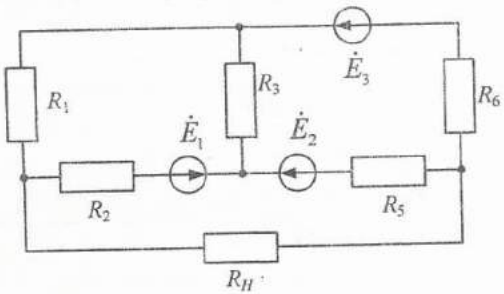


Fig. 8.12

The parameters of the circuit elements are: $R_1 = R_6 = 200\ \Omega$; $R_2 = R_5 = 500\ \Omega$; $R_3 = R_4 = 1\ k\Omega$; $E_1 = E_3 = 10$ V; $E_2 = 2$ V.

Questions for self-checking

1. Give the definition of the inductively coupled circuit. What are the magnetic fluxes and magnetic flux linkages of inductively coupled circuits? Give the definitions of fluxes and flux linkages of self-induction, mutual induction, leakage and the full flux and magnetic-flux linkage coils.

2. What is inductance? What kinds of inductances are there in inductively coupled coils? What are the degree of coupling and the coefficient of coupling between coupled coils?

3. What connection of magnetically coupled coils is called aiding and what is called opposing? What are like and unlike connections of magnetically coupled coils?

4. Write the equation for a series connection of magnetically coupled coils in complex form. Write the expression for the equivalent inductance of two magnetically coupled coils with the aiding and opposing connections. Explain basic variometer design and operation.

5. Build a vector diagram of the currents and voltages of two magnetically coupled coils with the aiding and opposing connections. Explain the phenomenon of a capacitive effect. What are the conditions for its arising?

6. Write the equation for a parallel connection of magnetically coupled coils in complex form. Write the expression for the equivalent inductance of two magnetically coupled coils with the aiding and opposing connections. Build vector diagrams for the currents and voltages for such connections.

7. Present the idea of an ideal and a real transformer. Write the equation for the two windings of a transformer. Consider an ideal and a real transformer, write the expression for the transformation coefficient, write the expression for the input resistance of a transformer.

8. Draw the equivalent circuit of a double-wound transformer whose coils are not magnetically coupled. Build a vector diagram of the transformer.

Problems

1. The circuit shown in Fig. 9.10 has the following elements:

$$R = 5\ k\Omega;\ C = 0{,}5\ \text{nF};\ L_1 = 3\ \text{mH};\ L_2 = 5\ \text{mH};$$

$$M = 2\ \text{mH};\ E = 10\ \text{V};\ \omega = 10\ \text{s}^{-1}.$$

Determine the complex input impedance of the circuit on the terminals $1 - 1'$ and the instantaneous value of the current.

Check whether the power balance holds.

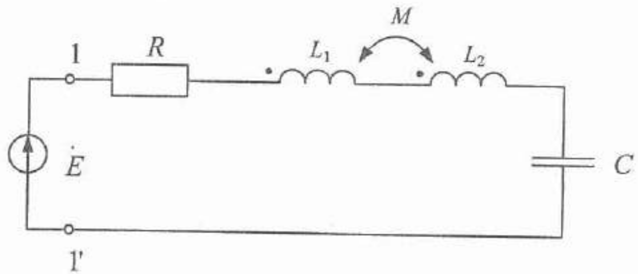


Fig. 9.10

2. Set up electrical balance equations to determine the branch current of the circuit (Fig. 9.11).

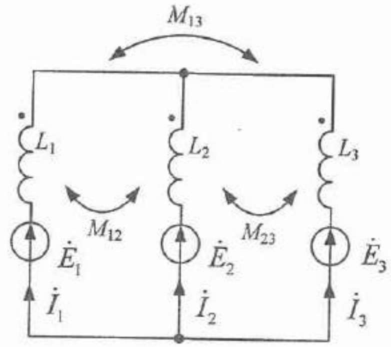


Fig. 9.11

3. Calculate the input impedance of the circuit (Fig. 9.12)

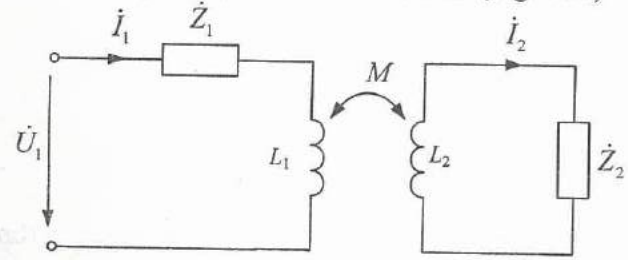


Fig. 9.12

4. Determine the input impedance of the circuit (Fig. 9.13) if

$C_1 = C_2 = 1$ nF; $L_1 = L_2 = 1$ mH; $R_1 = R_2 = 20\ \Omega$; $M = 48$ mH;
$\omega_1 = 10^6\ \text{s}^{-1}$; $\omega_2 = 1{,}02 \cdot 10^6\ \text{s}^{-1}$.

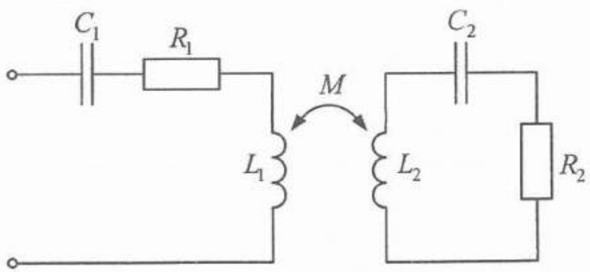


Fig. 9.13

5. A symmetric linear transformer having the parameters $L = 4{,}2$ mH; $R = 1{,}2\ \Omega$; $M = 1{,}8$ mH, is loaded to the capacity $C = 3{,}2$ mcF. Find the parameters of the series equivalent circuit of a transformer with respect to the input terminals at the frequency $f = 1$ kHz.

6. Determine the input impedance of an ideal transformer loaded to the impedance $Z_l = 20e^{j60^\circ}\ \Omega$.

The number of turns of primary winding $W_1 = 400$, secondary winding $W_2 = 40$. Find the complex RMS current I_1 and the voltage U_1 at the input terminals of the transformer if the current through the secondary winding is $I_2 = 0{,}12e^{j30^\circ}$ A..

Solution

The complex voltage transfer ratio is:

$$K(j\omega)=\frac{\dot{U}_2}{\dot{U}_1}=\frac{I_1R_2}{I_2(R_2+Z_{LR})}=\frac{R_2}{R_2+Z_{LR}};\ Z_{LR}=\frac{R_1j\omega L}{R_1+j\omega L}.$$

Then

$$K_u(j\omega)=\frac{R_2}{R_2+\dfrac{R_1j\omega L}{R_1+j\omega L}}=\frac{R_2(R_1+j\omega L)}{R_1R_2+R_2j\omega L+R_1j\omega L}$$

$$=\frac{R_1R_2+R_2j\omega L}{R_1R_2+(R_1+R_2)j\omega L}=\frac{1+j\omega\dfrac{L}{R_1}}{1+j\omega L\dfrac{R_1+R_2}{R_1R_2}}=\frac{1+j\omega\tau_1}{1+j\omega\tau_2}=$$

$$=\frac{\sqrt{1+(\omega\tau_1)^2}e^{j\mathrm{atan}\omega\tau_1}}{\sqrt{1+(\omega\tau_2)^2}e^{j\mathrm{atan}\omega\tau_2}}=\frac{\sqrt{1+(\omega\tau_1)^2}}{\sqrt{1+(\omega\tau_2)^2}}e^{j\mathrm{atan}\omega\tau_1-j\mathrm{atan}\omega\tau_2}.$$

Here: $\tau_1=\dfrac{L}{R_1}$; $\tau_2=\dfrac{L(R_1+R_2)}{R_1R_2}$ — time constants.

Hence, the AFC and PhFC are:

$$K_u(\omega)=\frac{\sqrt{1+(\omega\tau_1)^2}}{\sqrt{1+(\omega\tau_2)^2}};\ \varphi(\omega)=\mathrm{atan}\omega\tau_1-\mathrm{atan}\omega\tau_2.$$

The graphs of $K_u(\omega)$ — AFC and of $\varphi(\omega)$ — PhFC are shown in Fig. 10.1, *b*, *c*.

Example 2

Determine the AFC and PhFC of the complex voltage transfer ratio of the voltage divider whose circuit is given in Fig. 10.15.

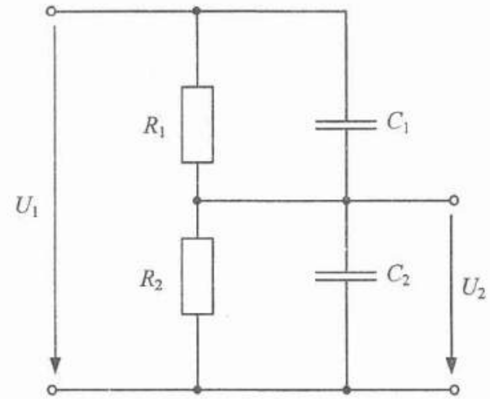


Fig. 10.15

Solution

The complex voltage transfer ratio is:

$$K_U(j\omega)=\frac{U_2}{U_1}=\frac{\Delta_{12}}{\Delta_{11}}.$$

Here Δ_{12}, Δ_{11} are algebraic adjuncts to the element of the 1-st row and the 2-nd column (for Δ_{12}) and of the 1-st row and the 1-st column (for Δ_{11}) of the node conductance matrix Y (NCM) for the circuit of Fig. 10.15.

The NCM is set up regarding the independent nodes 1 and 2.

We get:

$$Y=\begin{vmatrix}\dfrac{1}{R_1}+j\omega C_1 & -\dfrac{1}{R_1}-j\omega C_1\\ -\dfrac{1}{R_1}-j\omega C_1 & \dfrac{1}{R_1}+\dfrac{1}{R_2}+j\omega C_1+j\omega C_2\end{vmatrix}.$$

The algebraic adjuncts are:

$$\Delta_{12}=(-1)^{1+2}\left(-\frac{1}{R_1}-j\omega C_1\right)=\frac{1}{R_1}+j\omega C_1;$$

$$\Delta_{11}=(-1)^{1+1}\left(\frac{1}{R_1}+\frac{1}{R_2}+j\omega C_1+j\omega C_2\right)=\frac{1}{R_1}+\frac{1}{R_2}+j\omega C_1+j\omega C_2.$$

Then

$$K_u(j\omega)=\frac{\dfrac{1}{R_1}+j\omega C_1}{\dfrac{1}{R_1}+\dfrac{1}{R_2}+j\omega C_1+j\omega C_2}=\frac{1+j\omega R_1C_1}{R_1\left[\dfrac{R_1+R_2}{R_1R_2}+j\omega\left(C_1+C_2\right)\right]}=$$

$$=\frac{R_1R_2\left(1+j\omega R_1C_1\right)}{R_1\left[j\omega R_1R_2\left(C_1+C_2\right)+R_1+R_2\right]}=$$

$$=\frac{R_1R_2\left(1+j\omega R_1C_1\right)}{R_1\left(R_1+R_2\right)\left(1+j\omega\dfrac{R_1R_2}{R_1+R_2}\left(C_1+C_2\right)\right)}=$$

$$=\frac{R_2}{R_1+R_2}\cdot\frac{1+j\omega R_1C_1}{1+j\omega\dfrac{R_1R_2}{R_1+R_2}\left(C_1+C_2\right)}=\frac{R_2}{R_1+R_2}\cdot\frac{1+j\omega\tau_1}{1+j\omega\tau_{12}}.$$

Here $\tau_1=R_1C_1$; $\tau_{12}=\dfrac{R_1R_2\left(C_1+C_2\right)}{R_1+R_2}$ — time constants.

Hence the AFC and PhFC

$$K_u(\omega)=\frac{R_2}{R_1+R_2}\sqrt{\frac{1+\left(\omega\tau_1\right)^2}{1+\left(\omega\tau_{12}\right)^2}};\ \varphi(\omega)=\text{atan}\omega\tau_1-\text{atan}\omega\tau_2.$$

At $\omega=0$ the voltage transfer ratio $K_u(0)=\dfrac{R_2}{R_1+R_2}<1$. When $\omega\to\infty$, $K_u(\infty)=\dfrac{R_2}{R_1+R_2}\cdot\dfrac{\tau_1}{\tau_{12}}$.

If $\tau_1>\tau_{12}$, then the voltage transfer ratio increases from $\dfrac{R_2}{R_1+R_2}$, remaining less than one.

If $\tau_1<\tau_{12}$, the voltage transfer ratio decreases from $\dfrac{R_2}{R_1+R_2}$.

If $\tau_1=\tau_{12}$, the voltage transfer ratio does not depend on the frequency and is equal to $\dfrac{R_2}{R_1+R_2}$.

Questions for self-checking

1. Present the concept of the complex function of a circuit. Give the definition of the complex function of a circuit. For what kind of circuits are complex functions set up? Name the types of complex functions of a circuit.

2. Establish the relationship between the complex functions of a circuit and the circuit parameters in terms of the loop impedance matrix and the node conductance matrix of the circuit.

3. Give the expression for the amplitude, phase, real and imaginary frequency characteristics of a circuit. Explain the building of the amplitude-phase-frequency characteristic of a circuit. Present the concept of the logarithmic amplitude and phase frequency characteristics of a circuit and of their dimensions.

4. Give the expressions for the frequency characteristics of the simplest circuits of the first order and build their graphs. Give the idea of the amplitude-frequency characteristics of second- and third-order circuits.

Problems

1. Find the analytic expressions for the amplitude frequency characteristic (AFC) and the phase frequency characteristic (PhFC) of the voltage transfer ratio of the circuits (Fig. 10.16, *a*, *b*), where: $R_1=R_2=R$; $C_1=C_2=C$.

Give their qualitative graphic representation.

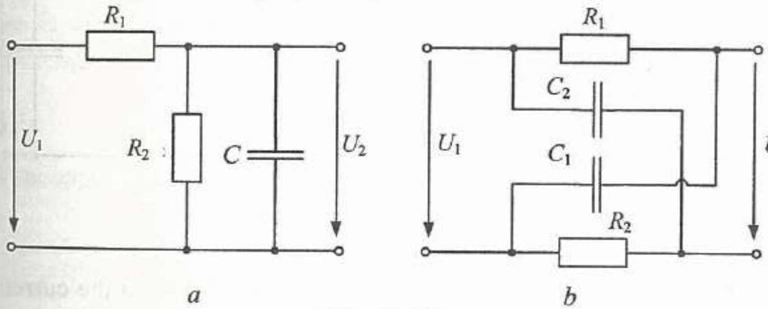


Fig. 10.16

2. Calculate the modulus and the argument of the complex input admittance of a parallel connection of a resistance and an inductance. Construct a hodograph of the input admittance if $R = 2\Omega$.

3. For the circuit (Fig. 10.17) determine the frequency at which the AFC of the voltage transfer ratio is maximum.

Calculate the phase shift of the input and output voltages at this frequency if $R_1 = R_2 = R = 1\ k\Omega$; $C_1 = C_2 = C = 1$ nF.

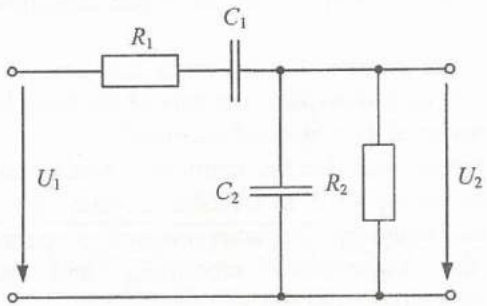


Fig. 10.17

4. Determine the frequency at which the phase shift between the input and the output voltages in the circuit (Fig. 10.18, *a*, *b*) is $-\frac{\pi}{2}$ and $-\pi$.

Find the modulus of the voltage transfer ratio at these frequencies if: $R_1 = R_2 = R_3 = R = 1\ k\Omega$; $C_1 = C_2 = C_3 = C = 1$ nF.

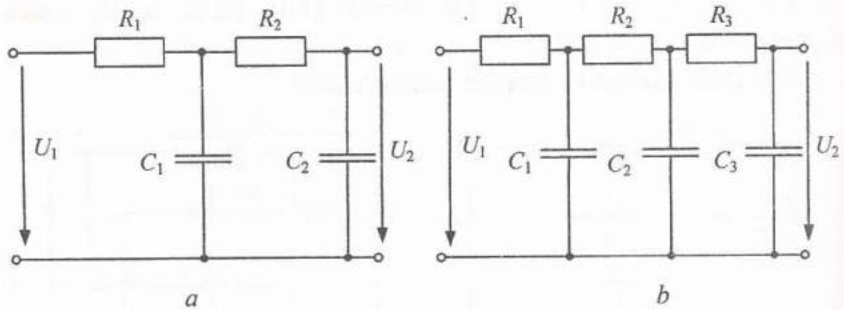


Fig. 10.18

5. Find the frequency at which the phase shift between the current in the resistance and the voltage at the input of the circuit is 180°.

The diagram of the circuit is shown in Fig. 10.19, where $L_1 = L_2 = L$.

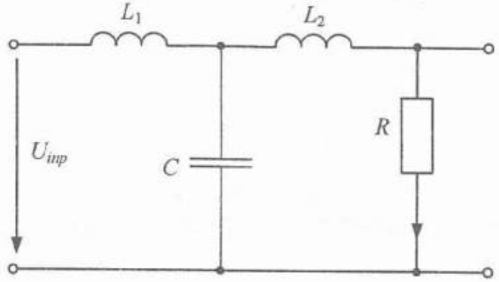


Fig. 10.19

Example 2

A series oscillatory circuit has the following elements: $L = 1{,}02$ mH; $C = 970$ pF. Find the bandwidth frequency limits for the two values of loss resistance: $R_1 = 10\ \Omega$; $R_2 = 350\ \Omega$.

Solution

The resonance frequency of the circuit is

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1{,}02\cdot 10^{-3}\cdot 970\cdot 10^{-12}}} = \frac{1}{\sqrt{989{,}4\cdot 10^{-15}}} = 1{,}003\cdot 10^6\ \text{s}^{-1}.$$

The characteristic impedance of the circuit:

$$\rho = \sqrt{\frac{L}{C}} = \sqrt{\frac{1{,}02\cdot 10^{-3}}{970\cdot 10^{-12}}} = 1{,}025\ k\Omega.$$

The quality factor of the circuit for the loss resistances R_1 and R_2:

$$Q_1 = \frac{\rho}{R_1} = \frac{1{,}003\cdot 10^6}{10} = 102{,}5;$$

$$Q_2 = \frac{\rho}{R_2} = \frac{1{,}003\cdot 10^6}{350} = 2{,}93.$$

The bandwidth of the circuit for the loss resistances R_1 and R_2:

$$2\Delta\omega_1 = \frac{\omega_0}{Q_1} = \frac{1{,}025\cdot 10^3}{102{,}5} = 9787\ \text{s}^{-1};$$

$$2\Delta\omega_2 = \frac{\omega_0}{Q_2} = \frac{1{,}025\cdot 10^3}{350} = 342{,}3\cdot 10^3\ \text{s}^{-1}.$$

The bandwidth frequency limits for the loss resistances R_1 and R_2:

$$\omega_{1b1} = \omega_0 - \frac{2\Delta\omega_1}{2} = 1{,}003\cdot 10^6 - \frac{9{,}987\cdot 10^3}{2} = 998\cdot 10^3\text{s}^{-1};$$

$$\omega_{1b2} = \omega_0 + \frac{2\Delta\omega_1}{2} = 1{,}003\cdot 10^6 + \frac{9{,}787\cdot 10^3}{2} = 1008\cdot 10^3\text{s}^{-1};$$

$$\omega_{2b1} = \omega_0 - \frac{2\Delta\omega_2}{2} = 1{,}003\cdot 10^6 - \frac{342{,}3\cdot 10^3}{2} = 832\cdot 10^3\text{s}^{-1};$$

$$\omega_{2b2} = \omega_0 + \frac{2\Delta\omega_2}{2} = 1{,}003\cdot 10^6 + \frac{342{,}3\cdot 10^3}{2} = 1174\cdot 10^3\text{s}^{-1}.$$

11. ***Questions for self-checking***

1. What is a series oscillatory circuit? Determine the resonance condition in a series oscillatory circuit. Write the expression for the resonance frequency, characteristic resistance, quality factor and attenuation of the circuit. Write the expression for the current and voltages in the circuit at resonance.
2. Describe the physical processes and determine the energy in a circuit and its elements at resonance. Express the quality factor of a series oscillatory circuit in terms of the powers at its elements at resonance.
3. Write the expressions for the complex input admittance of a series oscillatory circuit. Write the expressions for the normalized amplitude-frequency and normalized phase-frequency characteristics of a series oscillatory circuit; build graphs of these characteristics.
4. Write the expressions for the amplitude-frequency and phase-frequency characteristics of the voltage on the elements of a series oscillatory circuit and build their graphs.
5. Give the idea of the selectivity, bandwidth and shape factor of a series oscillatory circuit.

Problems

1. Determine the resonance frequency f_0, the characteristic resistance ρ, the quality factor Q, the attenuation d, and the bandwidth $2\Delta f_0$ of a series oscillatory circuit. The circuit elements are: $L = 180$ mcH; $C = 240$ pF; $R = 8{,}2\ \Omega$.
2. A series oscillatory circuit has the following parameters: the resonant frequency $f = 2$ MHz, the bandwidth $2\Delta f_0 = 16$ kHz and the loss resistance $R = 12\ \Omega$. Calculate the parameters of the reactive elements of the circuit.
3. A series oscillatory circuit with the elements $L = 2$ mH; $C = 1.5$ nF; $R = 32\ \Omega$ is connected to a source of harmonic EMF.

Determine the absolute Δf, relative δ, and generalized ξ detunings of the circuit at the frequencies: $f_1 = 100$ MHz; $f_2 = 92$ MHz; $f_3 = 88$ MHz. Find the modulus and the argument of the complex input impedance of the circuit at these frequencies.

4. A series oscillatory circuit consisting of an inductor $L = 220$ mcH and a capacitor $C = 535$ pF $(Q_C >> Q_L)$ and having the quality factor $Q = 120$ is connected to a source of energy with the internal resistance $R = 17\ \Omega$. Determine the resonance frequency and the bandwidth of the circuit.

5. Find the total impedance of a serial oscillatory circuit with the relative detuning $\delta_1 = \pm 0,01$; $\delta_2 = \pm 0,08$. The parameters of the circuit elements are: $L = 0,16$ mH; $C = 640$ pF; $R = 25\ \Omega$.

6. Calculate the bandwidth of a circuit (Fig. 11.9).

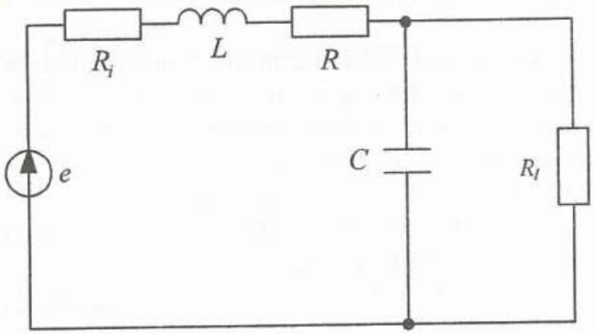


Fig. 11.9

The circuit elements are: $L = 1$ mH; $C = 1$ nF; $R = 5\ \Omega$; R_i=10 Ω; R_l=0.1 Ω.

Example 4

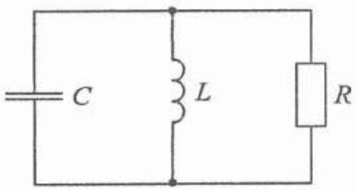


Fig. 11.21

Calculate the elements of a parallel oscillatory circuit (Fig. 11.21) with a bandwidth of $2\Delta f = 25$ kHz, the central bandwidth frequency (the resonance frequency) $f_0 = 1$ MHz and the characteristic impedance $\rho = \sqrt{\frac{L}{C}} = 75\ \Omega$.

Solution

The resonance frequency f_0 and the characteristic impedance ρ of the parallel oscillatory circuit are related as: $f_0 = \frac{1}{2\pi\sqrt{LC}}$; $\rho = \sqrt{\frac{L}{C}}$.

Solving these equations with respect to L and C, we get

$$C = \frac{1}{2\pi f_0 \rho} = \frac{1}{2\pi \cdot 1 \cdot 10^6 \cdot 75} = 2{,}12 \cdot 10^{-6} = 2{,}12 \text{ nF}$$

$$L = \frac{1}{2\pi f_0} = \frac{1}{2\pi \cdot 1 \cdot 10^6} = 11{,}9 \cdot 10^{-6} = 11{,}9 \text{ mcH.}$$

The quality factor is related with the bandwidth and the resonance frequency as $2\Delta f_0 = \frac{f_0}{Q}$. Hence, $Q = \frac{f_0}{2\Delta f_0} = \frac{1 \cdot 10^6}{25 \cdot 10^3} = 40$.

Questions for self-checking

1. What is a parallel oscillatory circuit? Define the resonance conditions in a parallel oscillatory circuit for different states in it at resonance.
2. Write the expressions for the complex input impedance of a parallel oscillatory circuit, build graphs of the amplitude-frequency and phase-frequency characteristics of a parallel oscillatory circuit.
3. Write the expression for the amplitude-frequency and phase-frequency characteristics in terms of current and voltage for a parallel oscillatory circuit connected to a real voltage source.
4. Give the idea of complex parallel oscillation circuits, name their types, write the expressions for the equivalent resistances of a complex parallel oscillatory circuit.
5. What is the inclusion coefficient of a complex parallel oscillatory circuit? What is the advantage of a complex parallel oscillatory circuit over a parallel oscillatory circuit of the first kind?
6. Give the definition of coupled oscillatory circuits. What types of coupling between circuits exist? What is the coupling degree between circuits? How are circuits subdivided by the coupling degree?

Problems

1. A parallel oscillatory circuit (see Fig. 11.20) is connected to a harmonic voltage source the frequency of which coincides with the resonant frequency of the circuit. The source parameters are: $E = 1$ V; $R = 120\ k\Omega$. Determine the RMS of the source current, the voltage in the circuit, and the currents in the capacitive and inductive branches of the circuit.

2. For the circuit considered in Fig. 11.20, determine the modulus $z(\omega)$ and the argument $\varphi(\omega)$ of the complex input impedance on the frequencies corresponding to the following values of the relative detuning:

$$\delta_1 = 10^{-2}; \delta_2 = 10^{-2};$$
$$\delta_3 = 10^{-1}; \delta_4 = 10^{-1}.$$

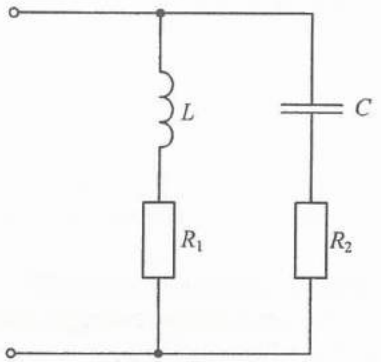


Fig. 11.22

3. Determine the current-related resonance frequency f_{0C}, the voltage-related resonance frequency f_{0v}, the quality factor Q, the characteristic impedance ρ, the inclusion coefficient p and the resonance resistance R_0 of a complex parallel oscillatory circuit (Fig. 11.22).

The circuit elements are:

$$L_1 = 150 \text{ mcH}; \ L_2 = 50 \text{ mcH}; \ C = 240 \text{ pF}; \ R_1 = 10 \ \Omega; \ R_2 = 4 \ \Omega.$$

4. Calculate current-related resonance frequency f_{0C}, the voltage-related resonance frequency f_{0v}, the characteristic impedance ρ, the quality factor Q, the inclusion coefficient m_c, the resonance resistance of the circuit R_0, the equivalent quality factor Q_C, the bandwidth $2\Delta f_0$ and the RMS voltage U_C in a circuit (Fig. 11.23) at resonant frequencies.

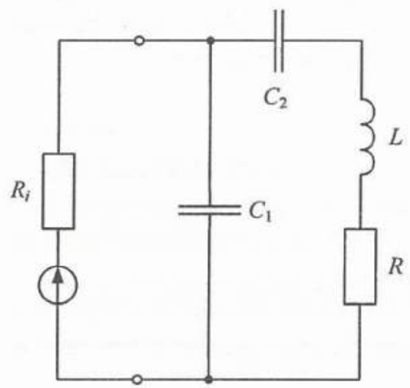


Fig. 11.23

The parameters of the circuit elements:

$$L = 220 \text{ mcH}; C_1 = 48 \text{ pF};$$
$$C_2 = 320 \text{ pF}; R_1 = 16 \ \Omega;$$
$$R_2 = 100 \ k\Omega.$$

Solution

The resonance frequency of the circuits:

$$\omega_{01} = \frac{1}{\sqrt{L_1 C_1}} = \omega_{02} = \frac{1}{\sqrt{L_2 C_2}} = \omega_0 = 2{,}5\times 10^{-6}\ \text{s}^{-1}.$$

Hence

$$C_1 = \frac{1}{\omega_0^2 L_1} = \frac{1}{\left(2{,}5\cdot 10^6\right)^2 \cdot 200\cdot 10^{-6}} = 800\text{pF};$$

$$C_2 = \frac{1}{\omega_0^2 L_2} = \frac{1}{\left(2{,}5\cdot 10^6\right)^2 \cdot 400\cdot 10^{-6}} = 400\text{pF}.$$

The Q-factors of the circuits

$$Q_1 = \frac{1}{R_1}\sqrt{\frac{L_1}{C_1}} = \frac{1}{10}\sqrt{\frac{200\cdot 10^{-6}}{800\cdot 10^{-12}}} = 50;$$

$$Q_2 = \frac{1}{R_2}\sqrt{\frac{L_2}{C_2}} = \frac{1}{20}\sqrt{\frac{400\cdot 10^{-6}}{400\cdot 10^{-12}}} = 50.$$

I.e. the Q-factors of the circuits are the same.

The mutual inductance M will be determined from the expression for the coupling coefficient:

$$k = \frac{M}{\sqrt{L_1 L_2}}.$$

Then

$$M = k\sqrt{L_1 L_2} = 0{,}03\sqrt{200\cdot 10^{-6}\cdot 400\cdot 10^{-6}} = 8{,}5\ \mu\text{H}.$$

The generalized detuning of the circuit:

$$\xi_1 = Q_1\left(\frac{\omega_{01}}{\omega_0} - \frac{\omega_0}{\omega_{01}}\right);\ \xi_2 = Q_2\left(\frac{\omega_{02}}{\omega_0} - \frac{\omega_0}{\omega_{02}}\right).$$

For $\omega_{01} = \omega_{02} = \omega_0$ the generalized circuit detuning $\xi_1 = \xi_2 = \xi = 0$.

The coupling factor:

$$A = \frac{x_c}{\sqrt{R_1 R_2}} = \frac{\omega_0 M}{\sqrt{R_1 R_2}} = \frac{2{,}5\cdot 10^6\cdot 8{,}8\cdot 10^{-6}}{\sqrt{10\cdot 20}} = 1{,}5.$$

The current flowing in the secondary circuit:

$$I_{m2} = \frac{E_1 A}{\sqrt{R_1 R_2}\sqrt{\left(1 + A^2 - \xi^2\right)^2 + 4\xi^2}}.$$

For $\xi = 0$:

$$I_2 = \frac{E_1 A}{\left(1 + A^2\right)\sqrt{R_1 R_2}} = \frac{100\cdot 10^{-6}\cdot 1{,}5}{\left(1 + 1{,}5^2\right)\sqrt{10\cdot 20}} = 3{,}26\ \mu\text{A}.$$

The voltage at the output of the amplifier:

$$U_2 = I_2\omega_0 L_2 = 3{,}26\cdot 10^{-6}\cdot 2{,}5\cdot 10^6\cdot 400\cdot 10^{-6} = 3{,}27\ \text{mV}.$$

The coupling factor A = 1,5, i.e. it is optimal. The system has three resonance frequencies. Two of them, at which the current I_{2mm} is maximum, are called the coupling frequencies.

Determine the attenuation of the circuits:

$$d=\frac{1}{Q}=\frac{1}{50}=0,02=d_1=d_2.$$

The critical coupling coefficient for a system of coupled circuits with three resonance frequencies is determined by the expression:

$$k_{cr}=\sqrt{\frac{d_1^2+d_2^2}{2}}=\sqrt{\frac{0,02^2+0,02^2}{2}}=0,02.$$

The coupling coefficient is specified as $k=0,03>k_{cr}$, i.e. there are the coupled frequencies ω_{01}, ω_{02} that are determined as:

$$\omega_{01}=\frac{\omega_0}{\sqrt{1+\sqrt{k^2-k_{cr}^2}}}=\frac{2,5\cdot 10^6}{\sqrt{1+\sqrt{0,03^2-0,02^2}}}=2,47\cdot 10\ \text{s}^{-1};$$

$$\omega_{02}=\frac{\omega_0}{\sqrt{1-\sqrt{k^2-k_{cr}^2}}}=\frac{2,5\cdot 10^6}{\sqrt{1-\sqrt{0,03^2-0,02^2}}}=2,53\cdot 10\ \text{s}^{-1}.$$

From the formula

$$\omega_{02}=\omega_0\left(1+\frac{1}{2Q}\xi\right)$$

we get the generalized circuit detuning

$$\xi=\frac{2Q(\omega_{02}-\omega_0)}{\omega_{02}}=\frac{2\cdot 50\cdot\left(2,53\cdot 10^6-2,5\cdot 10^6\right)}{2,53\cdot 10^6}=1,2.$$

The current I_{m2} in the secondary circuit for $\xi=1,2$ is

$$I_{m2}=\frac{E_1 A}{\sqrt{R_1R_2}\sqrt{\left(1+A^2-\xi^2\right)^2+4\xi^2}}=$$

$$=\frac{100\cdot 10^{-6}\cdot 1,5}{\sqrt{10\cdot 20}\sqrt{\left(1+1,5^2-1,2^2\right)^2+4\cdot 1,2^2}}=3,35\cdot 10^{-6}.$$

The voltage at the output of the amplifier for $\xi=1,2$ is $U_2=I_{m2}\omega_0 L_2=3,35\cdot 10^{-6}\cdot 2,5\cdot 10^6\cdot 400\cdot 10^{-6}=3,35$ mV.

Questions for self-checking

1. Analyze the processes in a system of two coupled circuits. What is the insertion resistance? Name their types.
2. What resonances occur in a system of two coupled oscillatory circuits? Name the resonance types. State the resonance conditions. What are the differences between simple and complex resonances?
3. Write the expressions for the frequency characteristics of the complex conductance transfer ratio for a system of two coupled oscillatory circuits in normalized form. Express their dependence on the coupling factor. Build graphs of the amplitude-frequency characteristics for different values of the coupling factor. Calculate the shape factor (bandwidth ratio) as a function the number of identical coupled oscillatory circuits.

Problems

1. For the circuit described in Example 11.3.1 determine the coupling frequencies f_1, f_2, the RMS value of the current in the secondary circuit, and the voltage on the capacitance C_2 at the resonant frequency and at the coupling frequencies.
2. Two coupled oscillatory circuits (Fig. 11.31) with the same parameters are tuned individually to the resonant frequency $f_0=0,8$ MHz. Find the values of C_1, C_2, C_{12} corresponding to the total resonance tuning with $L_1=L_2=L=120$ mkH; $Q_1=Q_2=Q=60$.
3. In a system of two identical oscillatory circuits with internal inductive coupling, determine the coupling factor in terms of coupling inductance for: a) $A=0,5$; b) $A=1$; c) $A=2,41$. The parameters of the circuit elements are:

$$L_{11}=L_{22}=0,8\ \text{mH};\ C_1=C_2=1,2\ \text{nF};\ R_1=R_2=40,8\ \Omega.$$

4. For a system of oscillatory circuits considered in Problem 3, determine the frequency range in which the current amplitude of the secondary circuit is reduced n-fold compared to its maximum value, where $n=\sqrt{2}$; $n=2$; $n=10$; $n=100$.